MTH 150 PRJT 5

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1 Section 5.1: Circles

1.1 Work from 1-2 (p. 343)

1.Find the distance between the points $(5, 3)$ and $(-1, -5)$
Answer:

$$dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$dist = \sqrt{(-1 - 5)^2 + (-5 - 3)^2} = 10$$

Reflection: By looking at example found at the beginning of the chapter the question was relatively straight forward. I had to use the distance formula, however I was mindful regarding how it works and visualized it in my mind as well as reflecting it on scrap paper to verify.

1.2 Work from 3-4 (p. 343)

3.Write an equation of the circle centered at $(8, -10)$ with radius 8.
Answer:

$$(x - h)^2 + (y - k)^2 = r^2$$

At center point $(8, -10)$ and radius 8:

$$(x - 8)^2 + (y + 10)^2 = 8^2$$

Reflection: This question didn't cause any issue for me as I recalled on high school knowledge to figure this out. I had to use the equation of a circle and substitute the values given.

1.3 Work from 5-6 (p. 343)

5.Write an equation of the circle centered at $(7, -2)$ that passes through $(-10, 0)$.
Answer:

Finding the Radius of the circle:

$$dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$dist = \sqrt{(-10 - 7)^2 + (0 - (-2))^2} = \sqrt{293}$$

Substituting known variables to the circle equation:

$$(x - 7)^2 + (y + 2)^2 = (\sqrt{293})^2$$

Reflection: Based on the prior question, this here incorporates similar aspects for me to understand.It was easier to complete due to the fact that I did questions similar to it's individual sections above.

1.4 Work from 7-8 (p. 343)

7.Write an equation for a circle where the points (2, 6) and (8, 10) lie along a diameter
Answer:

Choosing the two points to lay on the opposite sides on circumference of the circle. Therefore, Finding the center of the circle:

$$center = (\frac{2+8}{2}, \frac{6+10}{2}) = (5, 8)$$

Using one of the two given points and the newly found center to find the radius of the circle:

$$r = \sqrt{(8 - 5)^2 + (10 - 8)^2} = \sqrt{13}$$

Since center and radius is known, the equation can be found:

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 5)^2 + (y - 8)^2 = 13$$

Reflection: The question initially confused me, then I noticed that I am required to find 'a' equation of a circle whose diameter contains the given points. Having that aha moment helped me proceed to correctly solving the circle, so that the two points lay on the extreme ends of the diameter.

1.5 Work from 9-10 (p. 343)

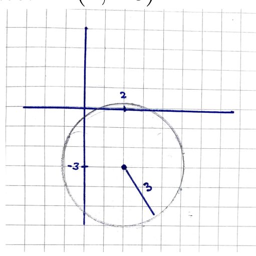
9. Write a equation of $(x-2)^2 + (y+3)^2 = 9$
   Answer: $(x-2)^2 + (y+3)^2 = 9$

Radius $= \sqrt{9} = 3$

$-h = -2 \rightarrow h = 2$

$-k = 3 \rightarrow k = -3$

$center = (2, -3)$



Reflection: I found this question easy to solve because I was given the variables to conduct what needed to be dissected, and the how to do aspect. Understanding that I broke down the problem into two radius and center .

1.6 Work from 11-12 (p. 343)

11.Find the y intercept(s) of the circle with center (2, 3) with radius 3.
   Answer:

Finding the equation of the circle:

$(x-h)^2 + (y-k)^2 = r^2$

$(x-2)^2 + (y-3)^2 = 3^2$

When $x = 0$:

$(0-2)^2 + (y-3)^2 = 9$

$(-2)^2 + y^2 - 6y + 9 = 9$

$y = 3 + \sqrt{5}$ and $y = 3 - \sqrt{5}$

Reflection: This was a relatively straight-forward question. It required me to find the equation and then continue with filling in the given variables to figure put the solution.

1.7 Work from 13-16 (p. 343)

13. At what point in the first quadrant does the line with equation $y = 2x + 5$ intersect a circle with radius 3 and center (0, 5)?

Answer:

Finding the equation of the circle:

$(x - h)^2 + (y - k)^2 = r^2$

$(x)^2 + (y - 5)^2 = 9$

Substituting $y = 2x + 5$ into the above equation:

$(x)^2 + (2x + 5 - 5)^2 = 9$

$5x^2 = 9$

$x = \pm\frac{3\sqrt{5}}{5}$

Substituting x into $y = 2x + 5$

$y = 2(\pm\frac{3\sqrt{5}}{5}) + 5$

$y = \frac{25\pm6\sqrt{5}}{5}$

Point of intersection in the first quadrant:

$(\frac{3\sqrt{5}}{5}, \frac{25\pm6\sqrt{5}}{5})$

Reflection: By reviewing the problem I saw the two possibilities here. To substitute x instead of y but that clearly ends up making it more tedious and complex. Substituting y cancels out terms and makes it simpler to get to the required answers.

1.8 Work from 17-21 (p. 343 - 345)

17.A small radio transmitter broadcasts in a 53 mile radius. If you drive along a straight line from a city 70 miles north of the transmitter to a second city 74 miles east of the transmitter, during how much of the drive will you pick up a signal from the transmitter?

Answer:

Imagine the circle's center placed on the origin of a Cartesian plane. The circle then as a center of $(0, 0)$ and a radius of 53.

$x^2 + y^2 = 53^2$

70 miles north from the center of the circle corresponds to a coordinate of $(0, 70)$ and 74 miles east from the center corresponds to a coordinate of $(74, 0)$. Using these two points, the gradient of line can be found and since north-axis corresponds to y-axis, the y-intercept of this line will be 70.

$c = 70$

$m = -\frac{70}{74}$

Therefore, $y = -\frac{35}{37}x + 70$

Using computer methods, the coordinates of the intersection of circle and the line approximates to:

(24.098, 47.205) and (45.794, 26.681)

Finding the distance:

$dist = \sqrt{(45.794 - 24.098)^2 + (26.681 - 47.205)^2} \approx 31.3 miles$

Reflection: Overall, and as you may have seen for the majority of the problems above I've changed my approach from the last chapter project. I want to breakdown my thought process in full detail in each question requires it. Here, I found the question semi tricky, however, placing the circle in the coordinate system in the above manner immediately simplified everything. As I progressed, finding the points of intersection became extremely cumbersome to be done by hand hence required me to use computer methods to get approximate values.

2 Section 5.2: Angles

2.1 Work from 5-6 (p. 359)

5.Convert the angle $\frac{5\pi}{6}$ from radians to degrees.

Answer:

$$\frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$$

Reflection: In order to solve this problem the conversion formula accurately solves the missing link. Understanding this gets me my answer.

2.2 Work from 11-14 (p. 359)

11.Find the angle between 0 and 2 in radians that is coterminal with the angle $\frac{26\pi}{9}$

Answer:

$$\frac{26\pi}{9} - 2\pi = \frac{8}{9}\pi$$

Reflection: The question puts the answer within the question, allowing the problem to be solved in one step.

2.3 Work from 15-24 (p. 359-360)

15.On a circle of radius 7 miles, find the length of the arc that subtends a central angle of 5 radians.

Answer:

$$s = r\theta$$

$$s = 5 * 7 = 35 miles$$

Reflection: This question was relatively straight forward and simple. By taking the variables in the question and plugging into the formula to solve the solution.

2.4 Work from 25-32 (p. 360-361)

25. A truck with 32-in.-diameter wheels is traveling at 60 mi/h. Find the angular speed of the wheels in rad/min. How many revolutions per minute do the wheels make?

Answer:

$60mi/hr \times \frac{1}{60} = 1mi/min$

1 mi converts to 63360 in, therefore the speed of the truck in inches:

$63360in/min$

Finding rev per min:

circumference $= 32 \times \pi$

rev/min $= \frac{63360}{32\pi} \approx 630rev/min$

Hence:

Angular velocity $= \frac{63360}{32\pi} \times 2\pi = 3960rad/min$

Reflection: At first I had trouble trying to find the angular speed; by researching the formula. I took an approach that reveals the revolution per minute using which I found the angular velocity.

26. A bicycle with 24-in.-diameter wheels is traveling at 15 mi/h. Find the angular speed of the wheels in rad/min. How many revolutions per minute do the wheels make?

Answer:

$15mi/hr \times \frac{1}{60} = \frac{1}{4}mi/min$

1 mi converts to 63360 in, therefore the speed of the truck in inches:

$63360 \times \frac{1}{4} = 15840in/min$

Finding rev per min:

circumference $= 24 \times \pi$

rev/min $= \frac{15840}{24\pi} \approx 210rev/min$

Hence:

Angular velocity $= \frac{15840}{24\pi} \times 2\pi = 1320rad/min$

Reflection: This question is identical to the last problem, solving this was simple to find.

3 Section 5.3: Points on Circles w/ Sine and Cosine

3.1 Work from 1-2 (p. 373)

1. Find the quadrant in which the terminal point determined by t lies if.

a. $sin(t) < 0$ and $cos(t) < 0$

b. $sin(t) > 0$ and $cos(t) < 0$
Answer:

a. sine and cosine are both negative in the third quadrant. Therefore the answer is quadrant three.

b. sine is positive in the 2nd quadrant and cosine is negative in the 2nd quadrant. Therefore the answer is quadrant two.

Reflection: By breaking down the problem in two sections, allowing me explain the concepts. It allows me to retain the problems concepts better.

3.2 Work from 3-4 (p. 373)

3. The point P is on the unit circle. If the y-coordinate of P is $\frac{3}{5}$ and P is in quadrant II, find the x coordinate.
Answer:

$y = sin(\theta) = \frac{3}{5}$

$adjacent = \sqrt{5^2 - 3^2} = 4$

$cos(\theta) = \frac{4}{5}$

$cosine$ is negative in the $2nd$ quadrant therefore:

x-coordinate $= \frac{-4}{5}$

Reflection: In this question, it required me to recall the triangle relationship between sine and cosine functions. A long shot but one that required khan academy explanation given the fact I was stuck for a portion solving.

3.3 Work from 5-8 (p. 373)

5. If $cos(\theta) = \frac{1}{7}$ and θ is in the 4th quadrant, find $sin(\theta)$
Answer:

$cos(\theta) = \frac{1}{7}$

$opposite = \sqrt{7^2 - 1^2} = 4\sqrt{3}$

$sin(\theta) = \frac{4\sqrt{3}}{7}$

sine is negative in the fourth quadrant, therefore:

$sin(\theta) = -\frac{4\sqrt{3}}{7}$

Reflection: This question is similar to the one above and the thought process behind it same.

6. If $cos(\theta) = \frac{2}{9}$ and θ is in the 1st quadrant, find $sin(\theta)$
Answer:

$cos(\theta) = \frac{2}{9}$

$opposite = \sqrt{9^2 - 2^2} = \sqrt{14}$

$sin(\theta) = \frac{\sqrt{14}}{7}$

sine is Positive in the first quadrant, therefore:

$sin(\theta) = \frac{\sqrt{14}}{7}$

Reflection: Similar to the last problem each problem is example for me to base my judgement and solve the problem.

3.4 Work from 11-12 (p. 373)

11. For each of the following angles, find the reference angle and which quadrant the angle lies in. Then compute sine and cosine of the angle.
a. $\frac{5\pi}{4}$

b. $\frac{7\pi}{6}$

c. $\frac{5\pi}{3}$

d. $\frac{3\pi}{4}$

Answer:

a. $\frac{5\pi}{4}$

$\pi < \frac{5\pi}{4} < \frac{3}{2}\pi$ therefore 3rd quadrant.

ref angle: $\frac{5\pi}{4} - \pi = \frac{1}{4}\pi$

$sin(\frac{1}{4}\pi) = \frac{\sqrt{2}}{2}$ sine is negative in the third quadrant, hence $-\frac{\sqrt{2}}{2}$

$cos(\frac{1}{4}\pi) = \frac{\sqrt{2}}{2}$ cosine is negative in the third quadrant, hence $-\frac{\sqrt{2}}{2}$

b. $\frac{7\pi}{6}$

$\pi < \frac{7\pi}{6} < \frac{3}{2}\pi$ therefore 3rd quadrant.

ref angle: $\frac{7\pi}{6} - \pi = \frac{1}{6}\pi$

$sin(\frac{1}{6}\pi) = \frac{1}{2}$ sine is negative in the third quadrant, hence $-\frac{1}{2}$

$cos(\frac{1}{6}\pi) = \frac{\sqrt{3}}{2}$ cosine is negative in the third quadrant, hence $-\frac{\sqrt{3}}{2}$

c. $\frac{5\pi}{3}$

$\frac{3}{2}\pi < \frac{5\pi}{3} < 2\pi$ therefore 4th quadrant.

ref angle: $2\pi - \frac{5\pi}{3} = \frac{1}{3}\pi$

$sin(\frac{1}{3}\pi) = \frac{\sqrt{3}}{2}$ sine is negative in the fourth quadrant, hence $-\frac{\sqrt{3}}{2}$

$cos(\frac{1}{3}\pi) = \frac{1}{2}$ cosine is positive in the fourth quadrant, hence $\frac{1}{2}$

d. $\frac{3\pi}{4}$

$\frac{1}{2}\pi < \frac{3\pi}{4} < \pi$ therefore 2nd quadrant.

ref angle: $\pi - \frac{3\pi}{4} = \frac{1}{4}\pi$

$sin(\frac{1}{4}\pi) = \frac{\sqrt{2}}{2}$ sine is positive in the second quadrant, hence $\frac{\sqrt{2}}{2}$

$cos(\frac{1}{4}\pi) = \frac{\sqrt{2}}{2}$ cosine is negative in the second quadrant, hence $-\frac{\sqrt{2}}{2}$

Reflection: After completing this problem, one of the most in depth questions I've solved in this project. By finding the reference angle I was able to explain such by drawing a unit circle helped reduce confusion.

3.5 Work from 13-14 (p. 374)

13. 3. Give exact values for $sin(\theta)$ and $cos(\theta)$ for each of these angles.
a. $-\frac{3\pi}{4}$

b. $\frac{23\pi}{6}$

c. $-\frac{\pi}{2}$

d. 5π

Answer:

In the following answers, a coterminal angle between 0 and 2π will be found and the exact values will be obtained by using the unit circle.

a. $-\frac{3\pi}{4}$

$-\frac{3\pi}{4} + 2\pi = \frac{5\pi}{4}$

$sin(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2}$ and $cos(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2}$

b. $\frac{23\pi}{6}$

$\frac{23\pi}{6} - 2\pi = \frac{11\pi}{6}$

$sin(\frac{11\pi}{6}) = -\frac{1}{2}$ and $cos(\frac{11\pi}{6}) = \frac{\sqrt{3}}{6}$

c. $-\frac{\pi}{2}$

$-\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}$

$sin(\frac{3\pi}{2}) = -1$ and $cos(\frac{3\pi}{2}) = 0$

d. 5π

$5\pi - 4pi = \pi$

$sin(\pi) = 0$ and $cos(\pi) = -1$

Reflection: At first, there was a bit of confusion, however, since I used coterminal angles before, I recalled it and found the relevant angles. Using those angles, I was able to answer as I've done in prior questions .

4 Section 5.1: The Trigonometric Functions

4.1 Work from 1-6 (p. 382)

1. If $\theta = \frac{\pi}{4}$ find exact values for $sec(\theta)$, $csc(\theta)$, $tan(\theta)$, $cot(\theta)$
   Answer:

$$sec(\theta) = \frac{1}{cos(\theta)} = \frac{1}{cos(\frac{\pi}{4})} = \sqrt{2}$$

$$csc(\theta) = \frac{1}{sin(\theta)} = \frac{1}{sin(\frac{\pi}{4})} = \sqrt{2}$$

$$tan(\theta) = tan(\frac{\pi}{4}) = 1$$

$$cot(\theta) = \frac{1}{\tan(\theta)} = \frac{1}{\tan(\frac{\pi}{4})} = 1$$

Reflection: In this question I found that using past problems to incorporate and understand this problem – allowed me to solve. Addittonally, recall the relationships between the functions.

4.2 Work from 9-14 (p. 382)

9. If $sin(\theta) = \frac{3}{4}$ is in quadrant II, $cos(\theta)$, $sec(\theta)$, $csc(\theta)$, $tan(\theta)$, $cot(\theta)$
   Answer:

$$sin(\theta) = \frac{3}{4}$$

$$adjacent = \sqrt{4^2 - 3^2} = \sqrt{7}$$

$$cos(\theta) = \frac{\sqrt{7}}{4} \text{ negative in II quadrant } -\frac{\sqrt{7}}{4}$$

$$sec(\theta) = \frac{4}{\sqrt{7}} \text{ negative in II quadrant } -\frac{4}{\sqrt{7}}$$

$$csc(\theta) = \frac{4}{3} \text{ positive in II quadrant}$$

$$tan(\theta) = \frac{sin(\theta)}{cos(\theta)} = \frac{3}{4} \times \frac{4}{\sqrt{7}} = \frac{3}{\sqrt{7}} \text{ negative in II quadrant } -\frac{3}{\sqrt{7}}$$

$$cot(\theta) = \frac{1}{tan(\theta)} = \frac{\sqrt{7}}{3} \text{ negative in II quadrant } -\frac{\sqrt{7}}{3}$$

Reflection: As stated this problem is similar to the last one, and solving wasn't difficult to understand..

4.3 Work from 17-26 (p. 383)

17. Simplify each of the following to an expression involving a single trig function with no fractions. $csc(t)tan(t)$

Answer:

Replacing the functions with its equivalents

$\frac{1}{sin(t)} \times \frac{sin(t)}{cos(t)}$

$\frac{1}{cos(t)} = sec(t)$

Reflection: Using trig function to solve understanding and writing out the solution was simple to see.

4.4 Work from 27-38 (p. 384)

27. Prove the identity $\frac{sin^2(\theta)}{1+cos(\theta)} = 1 - cos(\theta)$

Answer:

Using the identity $sin^2(\theta) + cos^2(\theta) = 1$

$\frac{1-cos^2(\theta)}{1+cos(\theta)} = 1 - cos(\theta)$

Using differences of two squares

$\frac{(1-cos(\theta))(1+cos(\theta))}{1+cos(\theta)} = 1 - cos(\theta)$

Reflection: The question was relatively straight forward. I had to recall a simple trig identity. Difference of two square rule simplified everything quite quickly.

5 Section 5.5: Right angle trigonometry

5.1 Work from 1-2 (p. 391)

1.In each of the triangles below, find $sin(A)$, $cos(A)$, $tan(A)$, $sec(A)$, $csc(A)$, $cot(A)$.

Answer:

Hypotenuse $= \sqrt{10^2 + 8^2} = 2\sqrt{41}$

$sinA = \frac{10}{2\sqrt{41}}$

$cosA = \frac{8}{2\sqrt{41}}$

$tanA = \frac{10}{8}$

$secA = \frac{2\sqrt{41}8}{}$

$cscA = \frac{2\sqrt{41}}{10}$

$cotA = \frac{8}{10}$

Reflection: As I'll mention in the questions below using a method familiar to me to solve was used to understand all that is mentioned here. .

5.2 Work from 3-8 (p. 391)

3. In each of the following triangles, solve for the unknown sides and angles.

Answer:

$sin(30) = \frac{7}{c}$

$c = \frac{7}{sin(30)} = 14$

$b = \sqrt{14^2 - 7^2} = 7\sqrt{3}$

Total internal angle of triangle is 180. Therefore:

$B = 180 - 30 - 90 = 60°$

Reflection: To solve I had to use my trig knowledge to figure out using SohCahToa. I recalled the simple geometric fact regarding the internal angle of a triangle which made it easier.

5.3 Work from 9-18 (p. 391-392)

9. A 33-ft ladder leans against a building so that the angle between the ground and the ladder is 80°. How high does the ladder reach up the side of the building?

Answer:

$sin(80) = \frac{x}{33}$

$x = 33sin(80) \approx 32.5ft$

Reflection: Based on this problem in order to understand it I had to draw a picture was drawn to follow through. After drawing it I understood to complete sin simply.

5.4 Work from 19-22 (p. 392-393)

19. Find the length x.

Answer:

Let the length of the adjacent side of the larger triangle be a and smaller triangle b.

$tan(39) = \frac{82}{a}$

$a = \frac{82}{tan(39)} \approx 101.3$

$tan(63) = \frac{82}{b}$

$b = \frac{82}{tan(63)} \approx 41.8$

$x = a + b$

$x = 101.3 + 41.8 = 143.1$

Reflection: This question was straight forward given my method to solve using SohCahToa. There was a repeat of the same method find the two independent sections of the side x

5.5 Work from 23-26 (p. 393-394)

23. A plane is flying 2000 feet above sea level toward a mountain. The pilot observes the top of the mountain to be 18° above the horizontal, then immediately flies the plane at an angle of 20° above horizontal. The airspeed of the plane is 100 mph. After 5 minutes, the plane is directly above the top of the mountain. How high is the plane above the top of the mountain (when it passes over)? What is the height of the mountain?

Answer:

The diagram consists of two triangles. Finding the hypotenuse of the larger triangle: The plane travels for 5 mins at a speed of 100mph, therefore the distance it moves, that is the hypotenuse we are finding is equal to:

$d = \frac{100}{60} \times 5 = \frac{25}{3} miles$

In feet:

$\frac{25}{3} \times 5280 = 44000ft.$

The horizontal distance the plane moved can be found. This is equal to the adjacent of both the triangles. Therefore:

Adjacent $= 44000 \times sin(20) \approx 41346ft$

Now we must find the opposite of the smaller rectangle. It tells us the height of the mountain above the initial position of the place.

$tan(18) = \frac{opp}{41346}$

$opp = tan(18) \times 41346ft \approx 13434$

Therefore the height of the mountain is approx $13434 + 2000 = 15434ft \approx 15000(2s.f)$

Finding the opposite of the big triangle gives us the height of the plane relative to its original position.

$opp = sin(20) \times 44000 \approx 15049ft$

Subtracting this value from the height of the mountain relative to the original position of the plane, i.e. 13434, gives the height of the plane above the mountain:

height above mountain $= 15049 - 13434 = 1615 \approx 1600ft(2s.f)$

Reflection: This question was not straight forward. I was initially confused with the 'airspeed' part and almost confused it with the horizontal speed. However, once the mental image was made the question quickly broke down to simple sections which required me to use trig functions and arithmetic.