MTH 150 PRJT 4

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October 2021

1 Section 4.1

1.1 Work from 7-12 (p. 263)

7. A population numbers 11,000 organisms initially and grows by 8.5% each year. Write an exponential model for the population.

$$P_0 = 11,000$$

$$b = 1 + 8.5\%$$

$$b = 1.085$$

$$P = P_0 b^t$$

$$P = 11,000(1 + 0.085)^t$$

$$\text{P} = 11,000(1.085)^t$$

Reflection: I found this problem easy to do given the fact we've done this problem in past chapter projects. Nothing difficult found while solving.

1.2 Work from 13-22 (p. 264)

13.Find a formula for an exponential function passing through the two points.

$(0, 6), (3, 750)$

$$y = ab^x$$

$$6 = ab^0$$

$$a = 6$$

$$y = 6b^x$$

$$750 = 6(b)^x$$

$$125 = b^3$$

$$b = \sqrt[3]{125}$$

$$b = 5$$

$$y = 6 \cdot 5^x$$

Reflection: As done in the previous chapters exponential functions stumps me temporarily. I say this because I need a refresher from an intro to exponential function Khan academy video(that is the name of it for reference). As I proceeded to watch the video I began copying the same methods over to the problem and solved everything accordingly.

1.3 Work from from 23-28 (p. 276)

23-25. Describe the long run behavior, as $x \to \infty$ and $x \to -\infty$ of each function.

23.$f(x) = -5\left(4^x\right) - 1$

As x approaches $-\infty, f(x)$ approaches the value of $y=-1$

As x approaches $\infty, f(x)$ approaches $-\infty$.

25. $f(x)=3\left(\frac{1}{2}\right)-2$

As x approaches $-\infty, f(x)$ approaches ∞.

As x approaches $-\infty, f(x)$ approaches $y=-2$

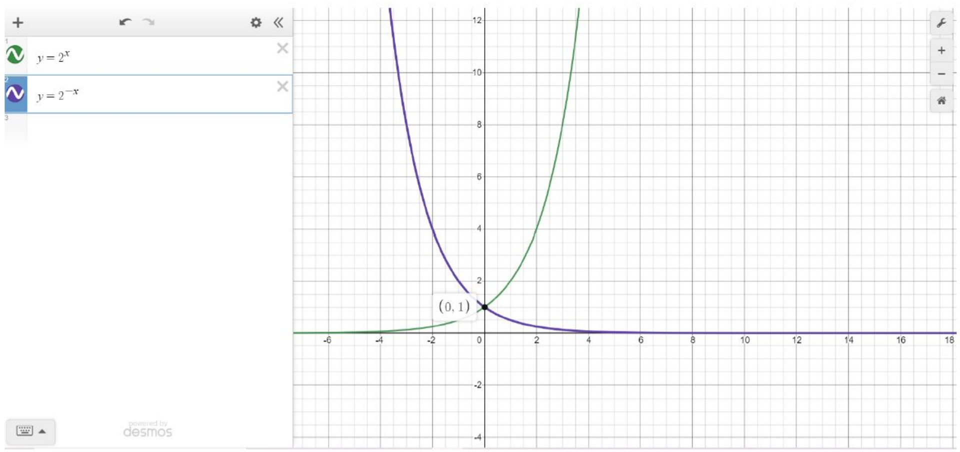
Reflection: This will encompass the reflection for both 23 25 because they are identical to one another. The biggest help to answer the problem was in class you had presented us an example to solve long run behavior for Chapter 3. After saying that it was the curve pattern flowing from left to right or reading a polynomial function you can solve. For 23. understanding the problem is divided into two pieces –¿ meaning -5(4^x)$will result a negative \infty$ and y value. While the second half being a mirror image, but f(x) is positive ∞ and the end result is toward negative ∞. For 25. The explanation above settles this here too.

2 Section 4.2

2.1 Work from 11-16 (p. 275)

11. Sketch a graph of each of the following transformation $f(x)=2^{-x}$

Graph:



Starting with the graph of $f(x) = 4^x$, find a formula for the function that results from.

Reflection: As you see in this problem I'm not using Mathematica. My laptop is unavailable to use because its being repaired by Microsoft. Desmos works better on iPad than online mathematica; full disclaimer before you're confused and comment this. Transparency is key thought to include that tid bit. By reflecting the original $f(x) = 2^{-x}$ coordinate I simply flipped to the opposite quadrant as the two possible transformations.

2.2 Work from 17-22 (p. 265)

17.Starting with the graph, find the formula that results from...

Shifting $f(x)$4 units upwards

$$f(x) = 4^{x-4}$$

Reflection: Based on the given variable and quickly looking at the question, I did anything my gut believed in. I solved it to prove myself right or wrong. By trying to solve I discovered the two function are connected to one another proving my answer was correct.

23. A radioactive substance decays exponentially. A scientist begins with 100 milligrams of a radioactive substance. After 35 hours, 50 mg of the substance remains. How many milligrams will remain after 54 hours?

$$\begin{gathered} y = ab^x \\ a = 100 \\ 50 = 100(b)^{35} \\ 0.5 = b^{35} \\ b = \sqrt[35]{0.5} \\ b \approx 0.98039 \\ y = ab^x \\ y = 100(0.98039)^x \\ f(x) = 100(0.98039)^x \\ f(54) = 100(0.98039)^{54} \end{gathered} \tag{1}$$

f(54) $\approx$ 34.32 grams

Reflection: In order to solve this problem I had to read it several times, although exponential I had a blank stare. Having an aha moment I realized that its similar to any other exponential problem and. had to dissect it into two parts: find radioactive decay of the substance for 35 hours, then, with the milligrams lost plug into an equation multiplying 100 and to the 54st power I got my answer.

3 Section 4.3

3.1 Work from 1-8 (p. 287)

Rewrite each equation in exponential form.

1. $\log_4(q) = m$

$$\log_4(q) = m$$

$$4^{\log_4(q)} = 4^m$$

$$q = 4^m$$

Reflection: Doing log here and problems below gave me no issues but a reminder. That practicing math problems done past, present, and eventually up to is always satisfying knowing I've understood a formula here.

Rewrite each equation in logarithmic form.

9. $4^x = y$

$$4^x = y$$

$$\log_4 4^x = \log_4 y$$

$$x = \log_4 y$$

$$\log_4(y) = x$$

Reflection: As stated above no challenges in solving only thing I needed to refresh myself was the code for log in LaTex.

Solve for x

17. $\log_3(x) = 2$

$$\log_3(x) = 2$$
$$3^{\log_3(x)} = 3^2$$
$$x = 3^2$$

$$\text{x} = 9$$

Reflection: Here I had to cancel one side of the function to accurately solve here. Not that there was a problem I had to self consciously ask myself does this make sense a few times just from the formation of the log. After looking at my old math notes from senior year I had a similar problem like this and connected the dots. Solve each equation for the variable.

Solve each equation for the variable.

41. $5^x = 14$

$$5^x = 14$$
$$\log_5 5^x = \log_5 14$$
$$x = \log_5 14/5$$

$$x \approx 1.64$$

Reflection: In this problem I had to work opposite of the prior two and include log into the given equation. Once I plugged in the log variable I did order of operations and divided the logs out and divided the 5 to the 14 to leave x to = 1.64

42. $3^x = 23$

$$\begin{aligned} 3^x &= 23 \\ \log_3 3^x &= \log_3 23 \\ x &= \log_3 23 \\ x &\approx 2.85 \end{aligned}$$

Reflection: As stated in the problem above because these are identical so solving this was smoothed out. Plugging in log and cancelling out the left side to leave x along and solve the rest on the right equals the answer listed.

65. The population of Kenya was 39.8 million in 2009 and has been growing by about 2.6% each year. If this trend continues, when will the population exceed 45 million?

$$\begin{aligned} P &= P_0(1 + \%\text{ increase })^t \\ P &= P_0(1 + 0.026)^t \\ P &= P_0(1.026)^t \\ P &= 39.8(1.026)^t \\ 45 &= 39.8(1.026)^t \\ \frac{45}{39.8} &= 1.026^t \\ \log\frac{45}{39.8} &= \log 1.026^t \\ t &= \frac{\log\frac{45}{39.8}}{\log 1.026} \\ t &\approx 4.78 \text{ years} \end{aligned}$$

Reflection: I believe it was in Chapter 3 and this chapter to solve for exponential growth over time. In this instance this problem incorporates log to solve for time equaling a set population number in Kenya. Using the equation above I took the given variables and solved ahead - the only tool I needed was my calculator to solve.

4 Section 4.4

4.1 Work from 1-16 (p. 298)

1. Simplify to a single logarithm, using logarithm properties.

$\log_3(28) - \log_3(7)$

$$\log_3(28) - \log_3(7) = \log_3 \frac{28}{7}$$

$$\log_3(28) - \log_3(7) = \log_3 4$$

Reflection: In order for me to solve this problem I had to remind myself logarithm properties and what it means. Two numbers with a base that equal the sum in this case $\log_3 \frac{28}{7}$

$. And the problem simply asked to simplify so I did by simplifying 28/7 and make \log_3 \frac{4}{}$

4.2 Work from 17-26 (p. 298)

17.Use logarithm properties to expand each expression.

$\log\left(\frac{x^{15}y^{13}}{z^{19}}\right)$

$$\log\left(\frac{x^{15}y^{13}}{z^{19}}\right) = \log x^{15} + \log y^{13} - \log z^{19}$$

$$\log\left(\frac{x^{15}y^{13}}{z^{19}}\right) = 15\log(x) + 13\log(y) - 19\log(z)$$

Reflection: The best way to explain this problem is how did I read and interpret this. Realizing that it shared the same qualities of the last problem I knew that the the variables and exponents needed to be separated to be seen as its most simplified form. And the only thing I wish was learning more about logs in the past.

4.3 Work from 27-48 (p. 299)

27. Solve each equation for the variable.

$4^{4x-7} = 3^{9x-6}$

$$\begin{aligned} 4^{4x-7} &= 3^{9x-6} \\ \log\left(4^{4x-7}\right) &= \log\left(3^{9x-6}\right) \\ (4x-7)\log(4) &= (9x-6)\log(3) \\ 4x\log(4) - 7\log(4) &= 9x\log(3) - 6\log(3) \\ 4x\log(4) - 9x\log(3) &= 7\log(4) - 6\log(3) \\ x(4\log(4) - 9\log(3)) &= 7\log(4) - 6\log(3) \\ x &= \frac{7\log(4) - 6\log(3)}{4\log(4) - 9\log(3)} \end{aligned}$$

x ≈ -0.72

Reflection:

5 Section 4.5

1. For each function, find the domain and the vertical asymptote.

$f(x) = \log(x-5)$

Domain:

$$\begin{aligned} x - 5 &> 0 \\ x &> 5 \end{aligned}$$

Domain: $(5, \infty)$

Vertical Asymptote:

$$x - 5 = 0$$

$$x = 5$$

2. $f(x) = \log(x+2)$

$$x + 2 > 0$$

$$x > -2$$

Domain: $(-2, \infty)$

Vertical Asymptote:

$$x + 2 = 0$$

$$x = -2$$

3. $f(x) = \ln(3 - x)$

$$3 - x > 0$$

$$-x > -3$$

$$x < 3$$

Domain: $(-\infty, 3)$

Vertical Asymptote:

$$3 - x = 0$$

$$-x = -3$$

$$x = 3$$

4. $f(x) = \ln(5 - x)$

$$5 - x > 0$$

$$-x > -5$$

$$x < 5$$

Domain: $(-\infty, 5)$

Vertical Asymptote:

$$5 - x = 0$$

$$-x = -5$$

$$x = 5$$

Reflection: Because all the problems were the same I simplify individual reflections into one. The problems here were another high school refresher solving domain and vertical asymptote. The take-away to best explain these problems to peer or you is to focus on the equation and take out the x and log variable to be less than 0 and simplify the function from there to get the domain.

6 Section 4.6

3. The half - life of Radium-226 is 1590 years. If a sample initially contains 200mg**, how many milligrams will remain after 1000 years?**

$$A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{1}{1990}}$$

$$A(1000) = 200 \left(\frac{1}{2}\right)^{\frac{1000}{1590}}$$

$$\mathrm{A(1000) = 129.33mg}$$

After 1000 years, there will 129.33mg **of Radium-226 remaining.**

4. The half - life of Fermium -253 **is 3 days. If a sample initially contains 100 mg, how many milligrams will remain after 1 week?**

$$A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{3}}$$

$$A(1000) = 100 \left(\frac{1}{2}\right)^{\frac{7}{3}}$$

$$\mathrm{A(1000) = 19.84mg}$$

After 1 week, there will 19.84 **mg of Fermium- 253 remaining.**

5. The half - life of Erbium- 165 is 10.4 **hours. After 24 hours a sample still contains** 2mg**. What was the initial mass of the sample, and how much will remain after another 3 days?**

$$A(24) = 2$$

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{1}{10.4}}$$

$$A(24) = A_0\left(\frac{1}{2}\right)^{\frac{24}{10.4}}$$

$$2 = A_0\left(\frac{1}{2}\right)^{\frac{24}{10.4}}$$

$$A_0 = \frac{2}{\left(\frac{1}{2}\right)^{\frac{24}{10.4}}}$$

$$A_0 = 9.9\text{mg}$$

$$A(72) = 9.9\left(\frac{1}{2}\right)^{\frac{72}{10.4}}$$

$$A(72) = 0.08157$$

A(72) = 0.082mg

After 3 days, there will 0.082 **mg of Erbium- 165 remaining.**

6. The half-life of Nobelium-259 is 58 minutes. After 3 hours a sample still contains 10mg. What was the initial mass of the sample, and how much will remain after another 8 hours?

$$A(180) = 10$$

$$A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{1}{58}}$$

$$A(180) = A_0 \left(\frac{1}{2}\right)^{\frac{180}{58}}$$

$$10 = A_0 \left(\frac{1}{2}\right)^{\frac{180}{58}}$$

$$A_0 = \frac{10}{\left(\frac{1}{2}\right)^{\frac{180}{58}}}$$

$$A_0 = 85.95\text{mg}$$

$$A(480) = 85.95 \left(\frac{1}{2}\right)^{\frac{480}{58}}$$

A(480) = 0.28mg

After 8 hours, there will be 0.28 mg of Nobelium -259 remaining.

Reflection: This reflection like some I'll merge based on the premise and similarity in this case chemistry. In all four problems I had to use an equation $A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{1}{x}}$. By plugging in the given variables of time in hours it solvable; the only translation to be done was if a problem had days instead, so converting 3 days to 72 hours. In the last two in order to solve the time frame given dissect and decide the important parts, how it begins and sample size, the half life of said object, and how much will be left after x hours. Assessing everything given allowed me to understand the problem swiftly.

29. The 1906 San Francisco earthquake had a magnitude of 7.9 **on the MMS scale. Later there was an earthquake with magnitude** 4.7 **that caused only minor damage. How many times more intense was the San Francisco earthquake than the second one?**

$$M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

$$M = \frac{2}{3}\log\left(\frac{S_1}{S_0}\right)$$

$$4.7 = \frac{2}{3}\log\left(\frac{S_1}{S_0}\right)$$

$$4.7\left(\frac{3}{2}\right) = \log\left(\frac{S_1}{S_0}\right)$$

$$7.05 = \log\left(\frac{S_1}{S_0}\right)$$

$$10^{7.05} = \frac{S_1}{S_0}$$

$$S_1 = 10^{7.05}S_0$$

$$7.9\left(\frac{3}{2}\right) = \log\left(\frac{S_2}{S_0}\right)$$

$$\frac{S_2}{S_1} = \frac{10^{11.85}S_0}{10^{7.05}S_0}$$

$$11.85 = \log\left(\frac{S_2}{S_0}\right)$$

$$10^{11.85} = \frac{S_2}{S_0}$$

$$S_2 = 10^{11.85}S_0$$

$$\frac{S_2}{S_1} = \frac{10^{11.85}S_0}{10^{7.05}S_0}$$

$$\frac{S_2}{S_1} = 10^{4.8}$$

$$S_2 = 63095.73S_1$$

30. The 1906 San Francisco earthquake had a magnitude of 7.9 on the MMS scale. Later there was an earthquake with magnitude 6.5 that caused less damage.How many times more intense was the San Francisco earthquake than the second one?

$$
\begin{aligned}
M &= \frac{2}{3}\log\left(\frac{S}{S_0}\right)\\
M &= \frac{2}{3}\log\left(\frac{S_1}{S_0}\right)\\
6.5 &= \frac{2}{3}\log\left(\frac{S_1}{S_0}\right)\\
6.5\left(\frac{3}{2}\right) &= \log\left(\frac{S_1}{S_0}\right)\\
9.75 &= \log\left(\frac{S_1}{S_0}\right)\\
10^{9.75} &= \frac{S_1}{S_0}\\
S_1 &= 10^{9.75}S_0\\
7.9\left(\frac{3}{2}\right) &= \log\left(\frac{S_2}{S_0}\right)\\
11.85 &= \log\left(\frac{S_2}{S_0}\right)\\
10^{11.85} &= \frac{S_2}{S_0}\\
S_2 &= 10^{11.85}S_0\\
\frac{S_2}{S_1} &= \frac{10^{11.85}S_0}{10^{9.75}S_0}\\
\frac{S_2}{S_1} &= 10^{2.1}\\
S_2 &= 125.89254S_1
\end{aligned}
$$

31. One earthquake has magnitude 3.9 on the MMS scale. If a second earthquake has 750 times as much energy as the first, find the magnitude of the second quake.

$$\begin{aligned}
M &= \frac{2}{3}\log\left(\frac{S}{S_0}\right) \\
3.9 &= \frac{2}{3}\log\left(\frac{S_1}{S_0}\right) \\
3.9\left(\frac{3}{2}\right) &= \log\left(\frac{S_1}{S_0}\right) \\
5.85 &= \log\left(\frac{S_1}{S_0}\right) \\
10^{5.85} &= \left(\frac{S_1}{S_0}\right) \\
\frac{S_2}{S_1} &= 750 \\
M &= \frac{2}{3}\log\left(\frac{750\left(10^{5.85}\right)S_0}{S_0}\right) \\
M &\approx 5.82
\end{aligned}$$

32. One earthquake has magnitude 4.8 on the MMS scale. If a second earthquake has 1200 times as much energy as the first, find the magnitude of the second quake.

$$\begin{aligned}
M =& \frac{2}{3}\log\left(\frac{S}{S_0}\right)\\
4.8 &= \frac{2}{3}\log\left(\frac{S_1}{S_0}\right)\\
4.8\left(\frac{3}{2}\right) &= \log\left(\frac{S_1}{S_0}\right)\\
7.2 &= \log\left(\frac{S_1}{S_0}\right)\\
10^{7.2} &= \left(\frac{S_1}{S_0}\right)\\
\frac{S_2}{S_1} &= 1200\\
S_2 &= 1200\left(10^{7.2}\right)S_0\\
M &= \frac{2}{3}\log\left(\frac{1200\left(10^{7.2}\right)S_0}{S_0}\right)\\
M &\approx 6.85
\end{aligned}$$

Reflection: For the four earthquake related questions a few key things were researched. One, MMS, magnitude moment scale, I needed context on how this formula works as well as the radiation decay in the prior questions. Second, the formula $M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$ from the beginning of the chapter with examples in detail by plugging in variables in two parts.

7 Section 4.7

Use regression to find an exponential function that best fits the data given.

9.

x	1	2	3	4	5	6
y	1125	1495	2310	3294	4650	6361

10.

x	1	2	3	4	5	6
y	643	829	920	1073	1330	1631

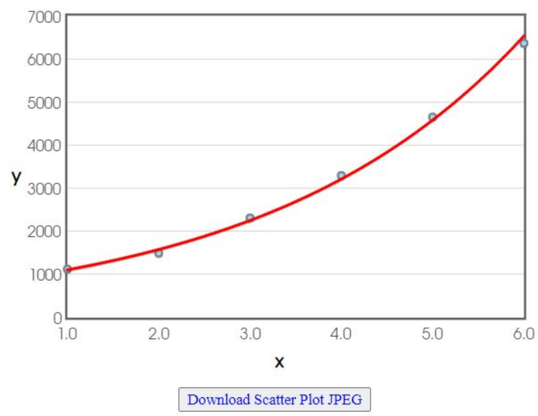
11.

x	1	2	3	4	5	6
y	555	383	307	210	158	122

12.

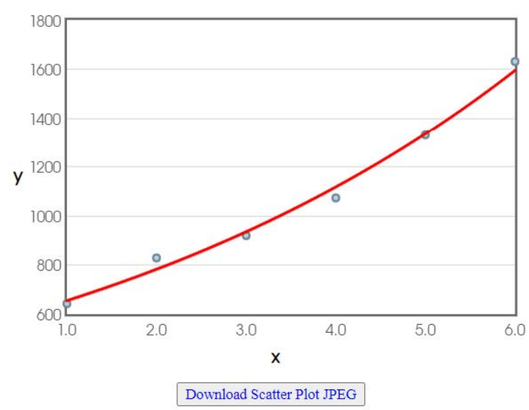
x	1	2	3	4	5	6
y	699	701	695	668	683	712

9.



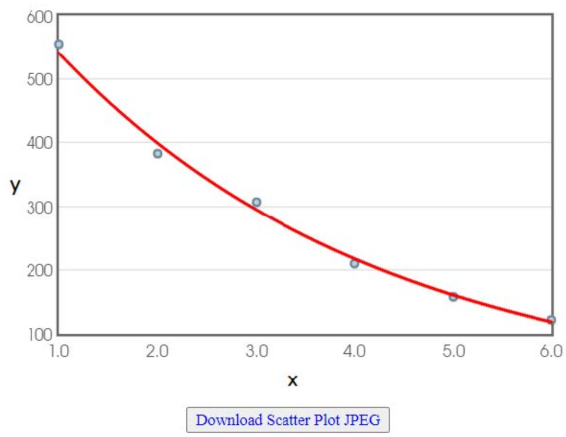
Regression Equation: $y = 776.6824 \cdot 1.426^x$
Correlation: $r = 0.9988$
R-squared: $r^2 = 0.9975$

10.



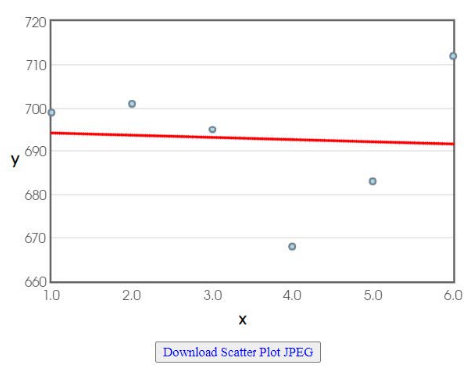
Regression Equation: $y = 548.5101 \cdot 1.1947^x$
Correlation: $r = 0.9946$
R-squared: $r^2 = 0.9892$

11.



Regression Equation: $y = 731.923 \cdot 0.7385^x$
Correlation: $r = -0.998$
R-squared: $r^2 = 0.996$

12.



Regression Equation: $y = 694.6267 \cdot 0.9993^x$
Correlation: $r = -0.0609$
R-squared: $r^2 = 0.0037$

Reflection: In the final four questions using regression to find exponential function by refreshing myself on the topic. At first I was confused on how to define this until I found an online scatter plot website to graph the data points. Looking on the output I can appreciate and better understand what was accomplished.