MTH 150 PRJT 3

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1 Section 3.1: Power Functions Polynomial Functions

1.1 Work from 1-8 (p.166)

Find the long run behavior of each function as $x \to \infty$ and $x \to -\infty$

1. $f(x) = x^4$

Answer:

As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to \infty$

Reflection: In order to best represent the problems question I had to understand how to dissect it. Understanding what long run behavior is writing out the answer was straightforward.

1.2 Work from 15-16 (p.166)

Find the degree and leading coefficient of each polynomial

15. $(2x + 3)(x - 4)(3x - 1)$

Answers

$(2x+3)(x-4)(3x-1) = (2x^2 - 5x - 12)(3x - 1) = 6x^3 - 13x^2 - 41x - 12$
Third degree, leading coefficient = 6

Reflection: The problem beneath this one is identical to the problem above with a few more details. One, both are looking for the highest degree found, in this case x$^3 isthegreatestdegreecomputed.Withthattheanswer.$

1.3 Work from 17-20 (p.166)

Find the long run behavior of each function as $x \to \infty$ and $x \to -\infty$

17. $-2x^4 - 3x^2 + x - 1$

Answers:

As $x \to \infty$**,** $f(x) \to -\infty$
As $x \to -\infty$ **,** $f(x) \to -\infty$

Reflection:Understanding that the answer was within the question by reading the context given I wanted to know more. Taking ten minutes to read Khan Academy articles and videos provided describing what is written above made sense.

1.4 Work from 21-22 (p.166)

21. What is the maximum number of $x-$intercepts and turning points for a polynomial of degree 5?

Answers:

degree $= 5$
Maximum $x-$**intercepts** $= 5$
Turning Points $=$ **degree** $-1 = 4$

Reflection: Using the just try it and the book methods of going about solving I gained the confidence to answer the following. Learning this again was eye opening to be able to appreciate the fact there's much to learn in this class. This especially. By realizing that the answer was in the question it made sense.

1.5 Work from 31-34 (p.166)

Find the vertical and horizontal intercepts of each function

32. $f(x) = 3(x+1)(x-4)(x+5)$

Answers:

$f(x) = 3(x+1)(x-4)(x+5)$

$x = 0$
$f(0) = 3(0+1)(0-4)(0+5)$
$f(0) = 3(1)(-4)(5)$
$f(0) = -60$

$y = 0$

$3(x+1)(x-4)(x+5)=0$
$3(x+1)=0 \Rightarrow x=-1$
$x-4=0 \Rightarrow x=4$
$x+5=0 \Rightarrow x=-5$

Horizontal intercepts: $(-1,0),(4,0),(-5,0)$
Vertical intercept: $(0,-60)$

Reflection: This problem here I had to look back at the prior chapter and watch a couple Youtube videos to understand the intercepts equations. As bland and generic as this sounds, I also incorporated the try something method for majority of this because the best thing I can do is attempt the problem.

2 Section 3.2: Quadratic Functions

2.1 Work from 7-12 (p.177)

For each of the follow quadratic functions, find a) the vertex, b) the vertical intercept, and c) the horizontal intercepts.

7. $y(x) = 2x^2 + 10x + 12$

Answers:

$y(x) = 2x^2 + 10x + 12$

$a = 2, b = 10, c = 12$

$x_v = \frac{-b}{2a} = \frac{-10}{(2)(2)} = -2.5$

$y_v = y(-2.5) = 2(-2.5)^2 + 10(-2.5) + 12 = 12.5 - 25 + 12 = -0.5$

$x = 0$
$f(0) = 2(0)^2 + 10(0) + 12$
$f(0) = 0 + 0 + 12$
$f(0) = 12$

$y = 0$
$2x^2 + 10x + 12 = 0$
$2(x + 2)(X + 3) = 0$
$x + 2 = 0 \Rightarrow x = -2$
$x + 3 = 0 \Rightarrow x = -3$

Vertex: $(-2.5, -0.5)$
Horizontal intercepts: $(-2, 0), (-3, 0)$
Vertical intercept: $(0, 12)$

Reflection: For this problem I recalled senior year when I discovered this method to solve versus what my teacher taught us, even got her to change her technique to solve vertex.

2.2 Work from 13-16 (p.177)

Rewrite the quadratic function into vertex form

16. $k(x) = 3x^2 - 6x - 9$

Answers:

$k(x) = 3x^2 - 6x - 9$

$a = 3, b = -6, c = -9$

$x_v = \frac{-b}{2a} = \frac{6}{(2)(3)} = 1$

$y_v = k(1) = 3(1)^2 - 6(1) - 9 = 3 - 6 - 9 = -12$

Vertex form:
$k(x) = 3(x - x_v) + y_v$
$k(x) = 3(x - 1)^2 - 12$

Reflection: Using the chapters resources as well as online examples. No issues in the solving process. Only gripe is learning all the LaTex commands is tedious and writing them into this compiler.

2.3 Work from 19-26 (p.178)

Write an equation for a quadratic with the given features

20. $x-$**intercepts** $(2, 0)$ **and** $(-5, 0)$**, and** y **intercept (0, 3)**

Answers:

x **intercepts:** $(2, 0), (-5, 0)$
$f(x) = a(x - 2)(x + 5)$

y **intercepts:** $(0, 3)$
$a(0 - 2)(0 + 5) = 3$
$a = \frac{3}{(-2)(5)} = -\frac{3}{10}$

Equation:
$f(x) = -\frac{3}{10}(x - 2)(x + 5)$

Reflection: As stated in previous inequality problem, no issues with solving or understanding this one. More practuce will help me solve faster for future examples and projects that may include this

2.4 Work from 27-30 (p.178)

28. A ball is thrown in the air from the top of a building. Its height, in meters above ground, as a function of time, in seconds, is given by $h(t) = -4.9t^2 + 24t + 8$

a. **From what height was the ball thrown?**

b. **How high above ground does the ball reach its peak?**

c. When does the ball hit the ground?

Answers:

a. $h(t) = -4.9t^2 + 24t + 8,\ t = 0$
$h(0) = -4.9(0)^2 + 24(0) + 8$
The ball was thrown from $h = 8$ **meter**

b. Find max from $t = -\frac{b}{2a}$

$a = -4.9, b = 24, c = 8$

$t = -\frac{24}{(2)(-4.9)} = \frac{24}{9.8} \approx 2.45$

$h(t) = -4.9t^2 + 24t + 8,\ t = 2.45$
$h(2.45) = -4.9(2.45)^2 + 24(2.45) + 8 = -29.412 + +58.8 + 8$
$h = 37.388$ **meter**

c. $h(t) = 0$
$-4.9t^2 + 24t + 8 = 0$
$a = -4.9, b = 24, c = 8$

$t_{1,2} = \frac{-24 \pm \sqrt{24^2 - 4(-4.9)(8)}}{2(-4.9)}$

$t_{1,2} = \frac{24 \pm \sqrt{732.8}}{9.8}$

$t_1 = 5.211$
$t_2 = -0.313$

The ball hit the ground when $t = 5.211$ **seconds.**

Reflection: Using the explanations provided in the chapter overview I was to solve the quadratic function without using mathematica for reference.

2.5 Work from 31-38 (p.179)

32. A box with a square base and no top is to be made from a square piece of cardboard by cutting 4 **in. squares out of each corner and folding up the sides. The box needs to hold** 2700 **in**^{3}**. How big a piece of cardboard is needed?**

Answers:

$4x^2 = 2700$

$x^2 = \frac{2700}{4} = 675$

$x = \sqrt{675}$
$x \approx 25.98$

Cardboard side $= 25.98 + 4 + 4 = 33.98$
So the size of the cardboard would be 33.98 **in by** 33.98 **in.**

Reflection: Thouroghly enjoyed this problem because of what I was solving for and knew how to complete. No problems solving, the book gave a weird example for me to understand, but understood it enough to create this as my answer.

3 Section 3.3: Graphs of Polynomial Functions

3.1 Work from 19-22 (p.191)

Solve each inequality.

19. $(x-3)(x-2)^2 > 0$

Answers:

$(x-3)(x-2)^2 > 0$
$x > 3$
$x > 2$
Solution: $x > 3$
Interval notation: $(3, \infty)$

Reflection: As stated in previous inequality problem, no issues with solving or understanding this one. More practice will help me solve faster for future examples and projects that may include this

3.2 Work from 19-22 (p.191)

Write an equation for a polynomial the given features.

31. Degree 3. **Zeros at** $x = -2$, $x = 1$, **and** $x = 3$. **Vertical intercept at** $(0, -4)$

Answers:

Zeros at $x = -2$, $x = 1$, **and** $x = 3$.
$(x+2)(x-1)(x-3) = 0$
Vertical intercept, $x = 0$, **at** $(0, -4)$ $a(0+2)(0-1)(0-3) = -4$
$6a = -4$

$a = -\frac{2}{3}$

Equation: $y = -\frac{2}{3}(x+2)(x-1)(x-3)$

Reflection: For this problem as well as the rest of the section I heavily used the books examples for polynomial equations. For me it was are refresher because I haven't done this since my junior year of high school. And practicing a few problems before solving definetly helped solve here.

3.3 Work from 51-52 (p.193)

51. A rectangle is inscribed with its base on the x axis and its upper corners on the parabola $y = 5 - x^2$. What are the dimensions of such a rectangle that has the greatest possible area?

Answers:

Set up the following variables:
$P(x, y)$ **is coordinate of the right hand corner**
A **is the area of rectangle**
P **lies on the parabola and** $y = 5 - x^2$**, so** $P = P(x, 5 - x^2)$
The base of the rectangle is half the distance between P **and the** $y-$**axis because it's a symmetry.**
Base $= 2x$ **and Height** $= y$

So the area of the rectangle is:
$A = B \times H$
$A = 2xy$
$A = 2x(5 - x^2)$
$A = 10x - 2x^3$

To maximise the Area as x **changes, we need to find** $\frac{dA}{dx}$

Differentiating the A
$\frac{dA}{dx} = 10 - 6x^2$

At critical point, $\frac{dA}{dx} = 0$

$10 - 6x^2 = 0$
$6x^2 = 10$
$x^2 = \frac{5}{3}$

$x = \pm\sqrt{\frac{5}{3}}$ **take the positive one because** x **is a non-negative**

$x \approx 1.29$

Base $= 2x = 2.58$ **and**
Height $= y = 5 - x^2 = 5 - \frac{5}{3} = 10/3 = 3.33$

Reflection: Being one of the biggest challenges in the section I required assistance from the book, Youtube videos, and mathematica to help me define the problem here. I took this as a learning lesson like we learn in class to throw a bunch of ideas and combine perspectives to create an answer like this here.

4 Section 3.4 Factor Theorem and Remainder Theorem

4.1 Work from 21-28 (p.202)

21. $x^3 - 6x^2 + 11x - 6, c = 1$
As one zero of this polynomial = 1, then (x-1) is a factor of the polynomial
We divide the polynomial $x^3 - 6x^2 + 11x - 6$ **by the factor x-1 by long division**
$x^2 - 5x + 6$

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x-1 — $x^3 - 6x^2 + 11x - 6$
$x^3 - x^2$

$-5x^2 + 11x - 6$
$-5x^2 + 5x$

6x-6
6x-6

0

Then $x^3 - 6x^2 + 11x - 6 = (x-1)(x^2 - 5x + 6)$
And we can factor $(x^2 - 5x + 6) = (x-2)(x-3)$

$So,$**x**$^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$

Then zeros of the polynomial are: 1, 2, 3.
And the factors of the polynomial are: $(x-1), (x-2), (x-3)$
Reflection:

25. $x^3 + 2x^2 - 3x - 6, c = -2$
As one zero of this polynomial = -2, then (x+2) is a factor of the polynomial
We divide the polynomial $x^3 + 2x^2 - 3x - 6$ **by the factor** $x + 2$ **by synthetic division**
-2 — 1 2 -3 -6
-2 0 6

---

1 0 -3 0

Then $x^3 + 2x^2 - 3x - 6 = (x+2)(x^2-3)$
And we can factor $(x^2-3) = (x-\sqrt{3})(x+\sqrt{3})$

So, $x^3 + 2x^2 - 3x - 6 = (x+2)(x-\sqrt{3})(x+\sqrt{3})$

Then zeros of the polynomial are: $-2, \sqrt{3}, -\sqrt{3}$**.**
And the factors of the polynomial are: $(x+2), (x-\sqrt{3}), (x+\sqrt{3})$
Reflection:

5 Section 3.5 Real Zeros of Polynomials

5.1 Work from 1-10 (p.209)

5. $f(x) = x^3 - 7x^2 + x - 7$

Using Cauchy's Bound tell us that the all real zeros exists in the interval $[-7/1 - 1, 7/1 + 1] = [-8, 8]$

ao=-7 ,factors of ao=7,-7,1,-1
an=1 ,factors of an=1,-1

The rational roots theorem tells us the possible rational zeros of the
polynomial are on the list ±1/1 ,±7/1 =±1,±7

Divide f(x) by (x-1) to check if 1 is a zero (using synthetic division)
1 — 1 -7 1 -7
1 -6 -5

---

1 -6 -5 -12
So 1 is not a zero.
Divide f(x) by (x-7) to check if 7 is a zero
7 — 1 -7 1 -7
7 0 7

---

1 0 1 0

So 7 is a real zero, and $f(x) = (x - 7)(x^2 + 1)$
$(x^2 + 1)$ **Can't be factorized.**
So there are 1 real zero and two complex zeros
Reflection:

5.2 Work from 11-32 (p.209)

14. $f(x) = x^3 + 4x^2 - 11x + 6$

ao=6,factors of $ao = 6, 1, 2, 3$
an=1,factors of $an = 1$

The rational roots theorem tells us the possible rational zeros of

the
polynomial are on the list $6/1, 3/1, 2/1, 1/1 = 6, 3, 2, 1$

Divide f(x) by (x-1) to check if 1 is a zero (using synthetic division)
1 — 1 4 -11 6
1 5 -6
————————————-
1 5 -6 0
So 1 is a zero.
Then $f(x) = (x-1)(x^2 + 5x - 6) = (x-1)(x-1)(x+6)$

So the real zeros are: 1,-6
Reflection:

6 Section 3.6 Complex Zeros

6.1 Work from 5-6 (p.217)

5. $(2+\sqrt{-12})/2$

$= (2+i\sqrt{3*4})/2 = (2+i2\sqrt{3}/2 = 1+i\sqrt{3}$
Reflection: Similar to the problem below in order to solve this problem I had to step back and theorize what will be the best method to simplify. Without hesitation broke down the square root twice into the simplest form as the answer.

6.2 Work from 19-24 (p.217)

22. $(6+4i)/i$
$= (6+4i)/i*(-i)/(-i) = (-6i-4i^2)/(-i^2) = (-6i+4)/1 = 4-6i$
Reflection: In this problem the simplest way that i interpreted to solve is solving simplest. Just as it sounds simplified the expression into a complex number by multiplying i. Then square rooting and subtracting the problem to get my answer of 6i.

6.3 Work from 25-42 (p.217)

29. $f(x) = x^3 + 6x^2 + 6x + 5$

ao=5,factors of ao=±5,±1
an=1,factors of an= ±1

The rational roots theorem tells us the possible rational zeros of the
polynomial are on the list ±5/1 ,±1/1 =±5,±1

Divide f(x) by (x-1) to check if 1 is a zero (using synthetic division)
1|1665
1713
$-------------------------------$
171318
So 1 is not a zero.

Divide f(x) by (x+5) to check if -5 is a zero
$-5|1665$
$-5-5-5$
$-------------------------------$
1110
So -5 is a zero
$f(x) = (x+5)(x^2+x+1)$

And we can factorize (x^2+x+1) **by the quadratic formula**

$x = (-1\sqrt{(1)^2 - 4(1)(1)})/2(1) = (-1\sqrt{-3})/2 = (-1)/2\sqrt{3}/2i$

So all the zeros are: $-5, (-1)/2 + \sqrt{3}/2i, (-1)/2 - \sqrt{3}/2i$

And the function is completely factorized to:
$f(x) = (x+5)(x+1/2-\sqrt{3}/2i)(x+1/2+\sqrt{3}/2i)$
Reflection: No issues present in this problem here. I laid my work as well as description in the problem to g step-by-step. Not only to help me but you reading this.

7 Section 3.7 Rational Functions

7.1 Work from 5-18 (p.217)

11. $a(x) = (x^2 + 2x - 3)/(x^2 - 1) = ((x + 3)(x - 1))/((x + 1)(x - 1)) = (x + 3)/(x + 1)$
Horizontal intercept is when a(x) = 0, x+3=0
x=-3
So the horizontal intercept are (-3,0)

Vertical intercept is when x = 0, a(0)=3/1=3
So the vertical intercept is (0,3)

Vertical asymptotes are when the denominator = 0
x+1=0
Vertical asymptote: x=-1

Horizontal asymptote is as x goes to inf., a(x)=1
Horizontal asymptote: y=1

No asymptotes.
Reflection: By evaluating the available variables given. And with the help of a friend I could piece together. One of the more challenging asymptotes to start but grew into the thought process to solve the others.

7.2 Work from 19-24 (p.235)

19.
- Vertical asymptotes at x=5, x=-5: the denominator = (x-5)(x+5)
- x intercepts at (2,0), (-1,0) : the numerator = (x-2)(x+1)

So $f(x) = a(x - 2)(x + 1)/(x - 5)(x + 5)$

But the y intercept is at (0,4)
$f(0) = (-2)/(-25)a = 4, so a = 50$

So $f(x) = 50 * (x - 2)(x + 1)/(x - 5)(x + 5) = 50 * (x^2 - x - 2)/(x^2 - 25)$
Reflection:Here, using mathematica to define vertical asymptotes I began mixing my knowledge of how to try different equations. My motivation came from the math video in class that

talked about trying anything. Thats what carried me and eventually solving the problem above.

7.3 Work from 39-44 (p.237)

41. $h(x) = (x^2 - x - 3)/(2x - 6)$

We do the long division:

1/2 x+1

$2x - 6 | x^2 - x - 3$
$x^2 - 3x$

2x-3
2x-6

3
So $h(x) = 1/2x + 1 + 3/(2x - 6)$

As x goes to inf $h(x) = 1/2 * (x + 1)$
The asymptote is y=1/2 x+1
Reflection: For this problem an outside friend assisted me in solving this as I was rusty at. Once he walked me through the steps to solve I slowly understood better but still have difficulty to grasp entirely.

7.4 Work from 45-49 (p.237-238)

45. $c(n) = (20 * 0.2)/(20 + n) = 4/(n + 20)$

c(10)=4/(10+20)*100=13.33

0.04=4/(n+20)

n+20=100
n=80 mL

As more and more added, the concentration approaches 0
Reflection: Another problem that allowed me to apply my physics background into real world context, here in this math problem.

Once I established all the variables to include writing out the equation was straightforward

8 Section 3.8 Inverses and Radical Functions

8.1 Work from 1-6 (p.245)

1. $f(x) = (x - 4)^2$
The domain in which the function is one-to-one and non-decreasing: (4,inf.)
We replace x and y with each other and get x in the left side
$x = (y - 4)^2$
$y - 4 = \sqrt{x}$
$y = \sqrt{x} + 4$
$f'(x) = \sqrt{x} + 4$
Reflection: No immediate problems found while solving for the inverse. As I state in the problem below I understand inverse and is a preffered section of math I'm good at.

8.2 Work from 7-16 (p.245)

11. $f(x) = 2/(x + 8)$
We replace x and y with each other and get x in the left side
$x = 2/(y + 8)$
$y + 8 = 2/x$
$y = 2/x - 8$
Reflection: In class last week we went over a similar problem. From that and notes I had from past years I was able to solve this without a lag. I like inverses and find them easy to solve.

8.3 Work from 17-20 (p.245)

17.
$v(215) = \sqrt{(20 * 215)} = \sqrt{4300} = 10\sqrt{43}$ **miles per hour**
v(215)=65.574 miles per hour Reflection: I found this problem in the subchapter to be mangeable based on some of the recent notes done in physics to help me. Using that knowledge I was able to identify the equation necessary to solve and plug the variables to create the one I have listed in the pdf.

8.4 Work from 21-24 (p.246-7)

21.
$y = ax^2$
$at x = 10, y = 10$
$10 = a * 100$
$a = 1/10$
$so y = 1/10x^2$
$when y = 5 : x^2 = 5 * 10 = 50$
$x = \sqrt{50} = 5\sqrt{2} = 7.071 feet$
so the width of the surface of water=2*7.071=14.142 feet

Reflection: As I did throughout most of this section, alot of youtube videos were required and the books example at the beginning. To get an idea where to go and found this one the most challenging problem to solve.